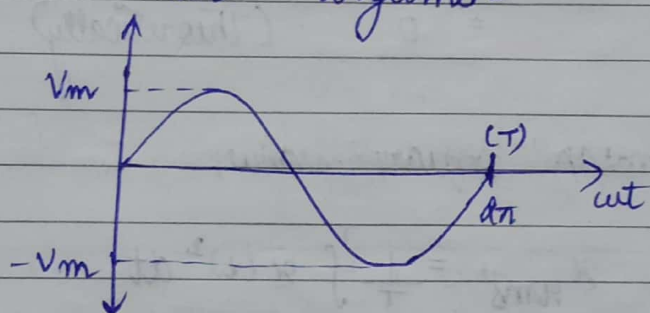
AC signals

So far we have studied opera^n of circuit ~~with~~ DC signal, now we will provide for sinusoidal signal as input.

$$V(t) = V_m \underbrace{\sin}_{\text{nature}}(\omega t)$$

V_m = Amplitude
ω = frequency
t = time
ωt = argument



$$\omega T = 2\pi$$

$$\omega = \frac{2\pi}{T}$$

$$\omega = 2\pi f$$

$$2\pi_{(rad)} = 360^\circ$$

$$1\ \text{rad} = 57.3^\circ$$

conversion factor

$$\boxed{\begin{aligned} \text{rad} &= \frac{\pi}{180} \times {}^\circ \\ {}^\circ &= \frac{180}{\pi} \times \text{rad} \end{aligned}}$$

Average value of Alternating signal :

In general, average value of general signal $x(t) = \frac{1}{T}\int_0^T x(t)\,dt$

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} V_m \sin d(\omega t)$$

$$= 0.63 V_m$$

for $V_{avg} = \frac{1}{\pi}\int_0^{2\pi} V_m \sin d(\omega t)$

$$= 0 \quad \text{(Theoretically)}$$

Root mean square value,

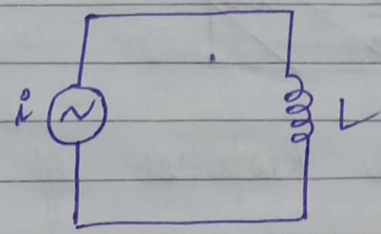
$$x_{rms} = \frac{1}{T}\int_0^T x(t)^2\,dt$$

$$V_{rms} = \frac{1}{2\pi}\int_0^{2\pi} (V_m \sin \omega t)^2\,d(\omega t)$$

$$= 0.707\, V_m$$

Response of AC signal in inductor or capacitor:

i)



(lags by 90°)

$$i = I_m \sin \omega t$$

$$V = L \frac{di}{dt}$$

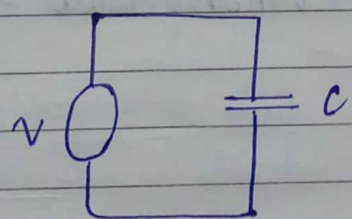
$$V = \omega L I_m \cos \omega t$$

$$= V_m \sin(\omega t + 90)$$

$$V_m = \omega L I_m$$

$$\boxed{\frac{V_m}{I_m} = \omega L}$$

ii)



(leads by 90°)

$$V = V_m \sin \omega t$$

$$i = C \frac{dv}{dt}$$

$$i = \omega C V_m \cos \omega t$$

$$i = I_m \cos \omega t$$

$$i = I_m \sin(\omega t + 90°)$$

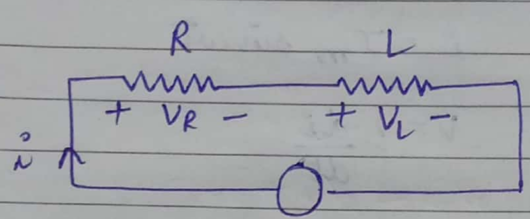
$$I_m = \omega C V_m$$

$$\boxed{\frac{1}{\omega C} = \frac{V_m}{I_m}}$$

$$\cancel{V_m = \omega L I_m}$$

$$\cancel{\boxed{\frac{V_m}{I_m} = \omega L}}$$

Sinusoidal Response of series R-L circuit:



$$V_R = iR = R I_m \sin \omega t$$

$$V_L = \omega L I_m \sin(\omega t + 90^\circ)$$

$$V = V_R + V_L$$

$$V_m \sin(\omega t + \underbrace{\theta}_{\text{phase shift}}) = R I_m \sin(\omega t) + \omega L I_m \sin(\omega t + 90^\circ)$$

$$V_m \sin \omega t \cos\theta + V_m \sin\theta \cos \omega t = R I_m \sin \omega t + \omega L I_m \sin \omega t \cos 90^\circ + \omega L I_m \sin 90^\circ \cos \omega t$$

here,

$$V_m \cos\theta = R I_m$$

$$V_m \sin\theta = \omega L I_m$$

$$V_m^2 = (R I_m)^2 + (\omega L I_m)^2$$

$$V_m = I_m \sqrt{R^2 + (\omega L)^2}$$

$$V_m = I_m \underbrace{Z}_{\text{impedence}}$$

$\rightarrow$ $R + j\omega L$

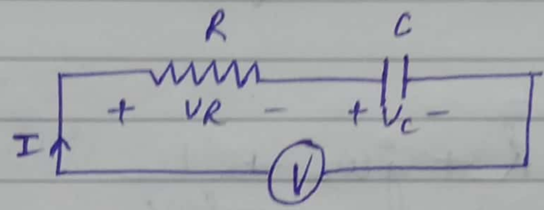
(lag by $\frac{\omega L}{R}$)

$\sqrt{R^2+(\omega L)^2} \angle \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$z = \sqrt{R^2+(\omega L)^2} \angle \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$I_m = \frac{V_m}{z} = \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Sinusoidal Response of series R-C circuit :



$$V = V_m \sin \omega t$$

$$V = I\left(R + \frac{1}{j\omega C}\right)$$

$$V = I\left[R - \frac{j}{\omega C}\right]$$

$$z = R - \frac{j}{\omega C}$$

$$z = \sqrt{R^2+\left(\frac{1}{\omega C}\right)^2} \angle -\tan^{-1}\frac{1}{\omega RC}$$

(current leads by $\frac{1}{\omega RC}$)

$$I = \frac{V}{z}$$

$$= \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2} \angle -\tan^{-1}\frac{1}{\omega RC}}$$

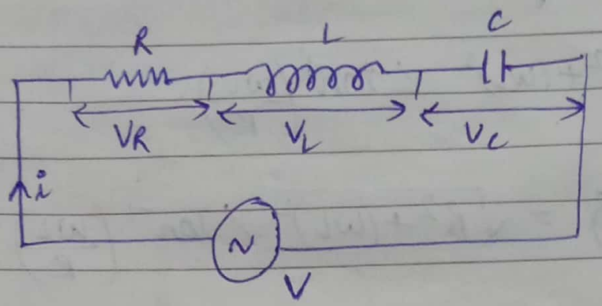
$$= \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \angle \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

$X_c = \frac{1}{\omega C}$

$= \frac{1}{0}$

$= \infty$

Series RLC circuit :



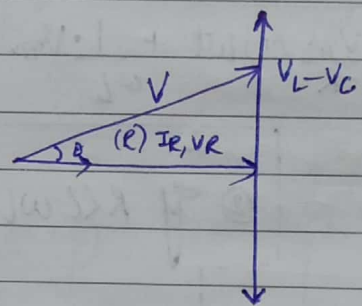
$$V = V_R + V_L + V_C$$
$$= iR + iX_L + iX_C$$
$$= i\left(R + j\omega L + \frac{1}{j\omega C}\right)$$

$$\left\{ \begin{array}{l} X_L = j\omega L = jX_L \\ X_C = \frac{1}{j\omega C} \approx \frac{-j}{\omega C} = -jX_C \end{array} \right.$$

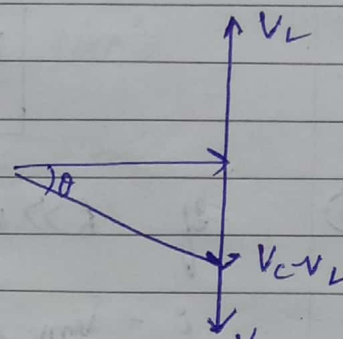
$$V = IZ$$
$$Z = R + j\omega L + \frac{1}{j\omega C}$$
$$Z = R + j\omega L - \frac{j}{\omega C}$$
$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$
$$\text{mag.} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$
$$\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

① circuit becomes inductive when $\omega L > \frac{1}{\omega C}$

② circuit becomes capacitive $\frac{1}{\omega C} > \omega L$

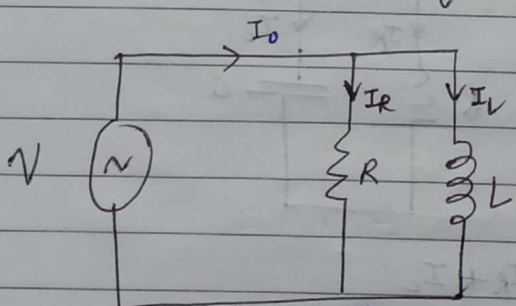


(inductive)



(capacitive)

Sinusoidal response of $||^{l}$ R-L ckt



$V = V_m \sin \omega t$

$I_0 = I_L + I_R$

$I_R = \frac{V_m \sin \omega t}{R} = \frac{V}{R}$; $I_L = \frac{V}{X_L}$

$$I_0 = \frac{V}{R} + \frac{V}{X_L}$$

$$I_0 = V\left(\frac{1}{R} + \frac{1}{j\omega L}\right)$$

opposite of impedance

Y admittance → $\left(I = \frac{1}{Y}\right)$

$$\boxed{Y = \frac{1}{R} + \frac{1}{j\omega L} = \frac{1}{R} - \frac{j}{\omega L}}$$

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Sinusoidal i/p , $v = v_m \cos \omega t$

$$I = I_R + I_L$$

$$I = \frac{v}{R} + \frac{1}{L}\int v\,dt$$

$$\boxed{I = \frac{V_m}{R}\cos \omega t + \frac{1}{\omega L}\cdot V_m \sin \omega t}$$

① if $R >> \omega L$

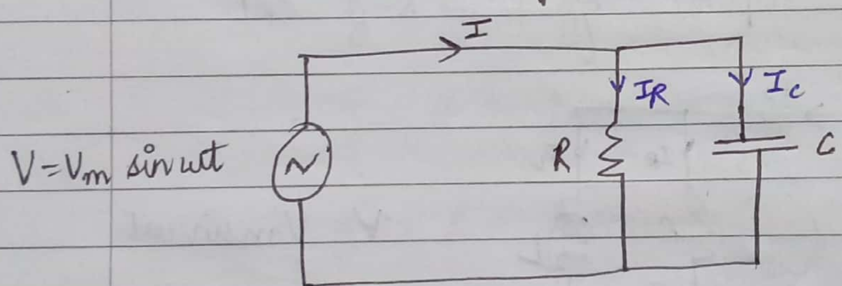
Phasor

$$i = \frac{V_m}{\omega L}\cos(\omega t - 90^\circ)$$

② if $R << \omega L$

$$i = \frac{V_m}{R}\cos \omega t$$

Sinusoidal series of $\|^{l}$ R-L circuit.



$$I = I_R + I_c$$

$$I = \frac{v}{R} + \frac{v}{\frac{1}{j\omega c}}$$

$$I = \frac{v}{R} + j v \omega c$$

$$I = I_R + I_c$$

$$= \frac{V_m \sin \omega t}{R} + c\frac{dv}{dt}$$

$$= \frac{V_m \sin \omega t}{R} + \omega c V_m \cos \omega t$$

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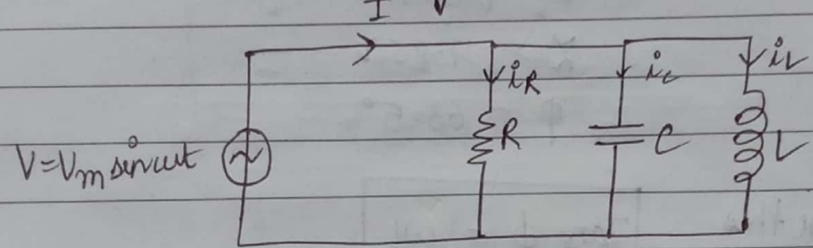
i) if $R \gg X_c$

$i = i_c = \omega C V_m \sin(\omega t + 90^\circ)$

ii) if $R \ll X_c$

$i \approx i_R \approx \frac{V_m}{R} \sin \omega t$

sinusoidal series of $||^l$ R-L-C circuit



$$I = i_R + i_C + i_L$$

$$I = \frac{V}{R} + \frac{V}{X_L} + \frac{V}{X_C}$$

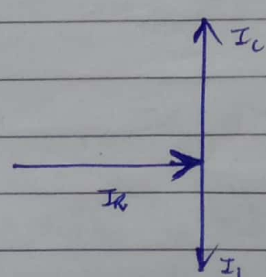
$$\cancel{I = \frac{V_m}{R} \sin \omega t}$$

$$I = \frac{V}{R} + \frac{V}{j\omega L} + \frac{V}{\frac{1}{j\omega C}}$$

$$I = \frac{V}{R} + \frac{V}{j\omega L} + V j\omega C$$

$$I = \frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt}$$

$$= \frac{V_m}{R}\sin\omega t - \frac{V_m}{L}\frac{\cos\omega t}{\omega} + \cancel{C}\, V_m \omega C \cos\omega t$$



(all values in rms)

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Q In a R-L circuit the inductance being 20 milli henry (mH), impedence 17.85Ω ϕ ~~angle of the lag~~ of the input current from the applied voltage. find the value of Ω & R.

$\omega = 796.875$ rad/s
$R = 8\Omega$

$L = 20mH$
$Z = 17.85\Omega$
$\phi = 63.5^\circ$

Use this $\boxed{\tan\phi = \frac{\omega L}{R}}$

Q Applied voltage is ‖^{l} RLC circuit is given by,

$v = 50\sin\left(5000t + \frac{\pi}{4}\right)V$
& $R = 20\,\Omega$
$L = 1.6\times10^{-3}H$
$C = 20\,\mu F$

find total current?

$I = 1.975\angle 18.5A$

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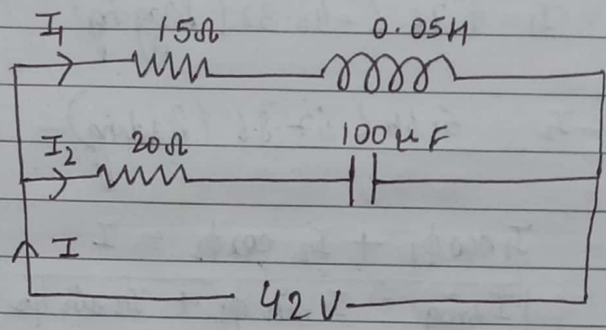
Q Determine rms value of current in each branch and total current of the circuit draw its phasor diagram.



$f = 50Hz$

$Z_1 = R_1 + j\omega L_1$

$= 15 + j(2\pi) \times 50 \times 0.05$

$= 15 + j(15.71)$

$Z_1 = \sqrt{(15)^2 + (15.71)^2}$

$= 21.72\Omega$

$\tan^{-1}\left(\frac{15.71}{15}\right) = \angle 46.32$ (lagging voltage)

$Z_2 = 20 + (-j) \frac{\times 1 \times 10^4}{2\pi \times 50 \times 100 \times 10^{-6}}$

$= 20 + (-j) \times \frac{100}{\pi}$

$= 20 - 31.81j$

$|Z_2| = 37.58$ $\angle 57.86°$ (leading)

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$$I_1 = \frac{V}{Z_1} = \frac{212}{21.72 \angle 46.32} \qquad I_2 = \frac{212}{37.58 \angle 57.86}$$

$I_1 = 9.76 \angle -46.32$ (lagging)

$I_2 = 5.64 \angle 57.86$ (leading)

$$I_1 \cos\phi_1 + I_2 \cos\phi_2 = I$$

$$\boxed{I_{img} = I_1 \sin\phi_1 + I_2 \sin\phi_2}$$

$$I = I_R + j\, I_{img}$$

$$\phi = \tan^{-1}\left(\frac{I_I}{I_R}\right)$$

$$I_{Real} = 9.76 \cos(46.32) + 5.64 \cos(57.86)$$
$$= 9.74$$

$$I_{img} = -2.28$$

$$\phi = \tan^{-1}\left(\frac{I_{img}}{I_R}\right) = -13.17^\circ$$

Reference line (which is common) in this case voltage is common.

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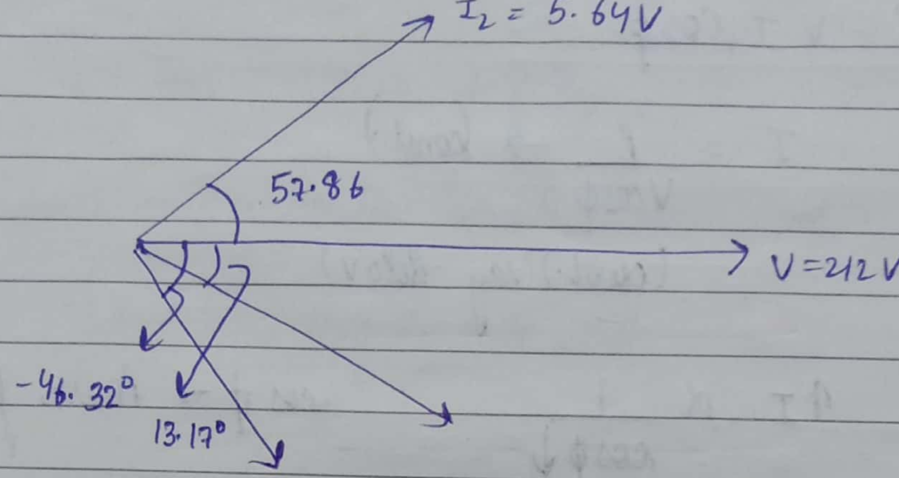
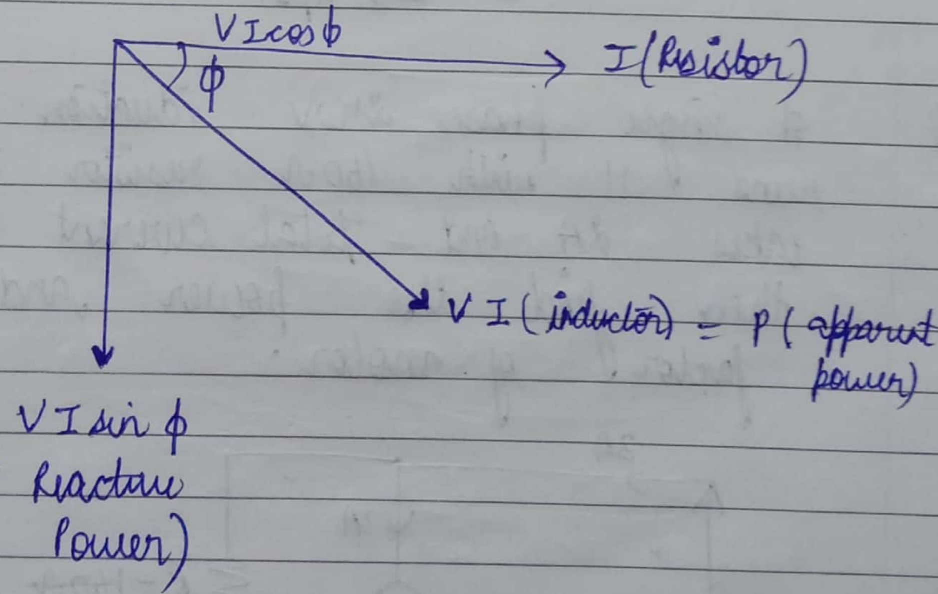


$\frac{1}{X_L}$ = susceptance

conductance + suspectanse = admittance (z^{-1})
$(X_C \text{ or } X_L)^{-1}$

Real / actual / active power



Resistor also has P = V I cos ϕ
But ϕ is 0

Real power → power consumed by load

Reactive power → not consumed by load.

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$P = VI\cos\phi$

$I = \frac{P}{V\cos\phi} \rightarrow$ (const)

(const.) (say 220V)

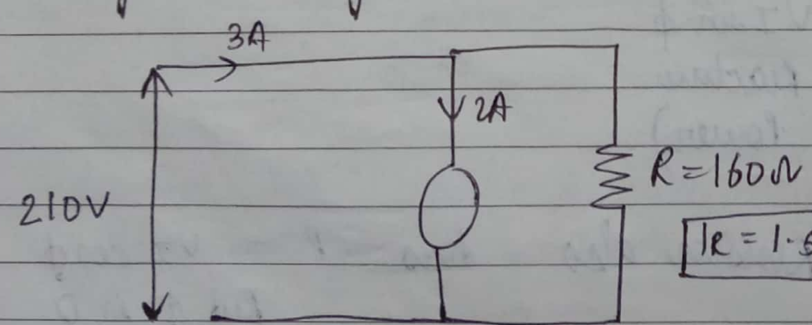
$\uparrow I \propto \frac{1}{\cos\phi \downarrow}$ $\cos\phi \rightarrow$ Power factor

if $\cos\phi < 1$, i increases (i.e. equip is drawing more current from source)

Real power consumed by inductor / capacitor = 0

$V I \underbrace{\cos\phi}_{\hookrightarrow \pi/2}$

Q A single phase 240V induction motor runs ‖ with 160Ω resistor if motor takes 2A and total current is 3A, then find the power and power factor of motor.



$\boxed{I_R = 1.5A}$

there will be lagging power factor in motor since more current is drawn than should be.

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$$I_m = I_{wt} + j I_\mu$$

$$I_m + I_R = 3A$$

$$\boxed{I_w + j I_u + I_R = 3}$$

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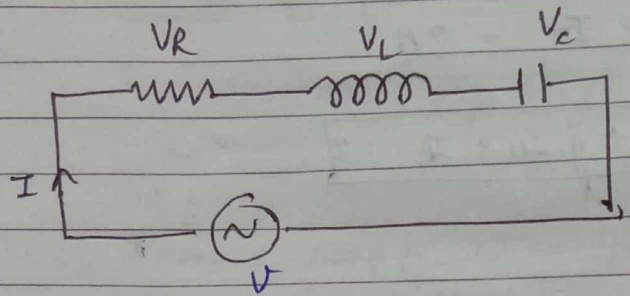
Series Resonance :

(1)



$$I = \frac{V}{Z}$$

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$Z = R + j(X_L - X_C)$$

$$I = \frac{V}{R + j(X_L - X_L)}$$

at resonance

$$X_L = X_C$$

$$I = \frac{V}{R}$$

(2) $\cos\phi = \frac{R}{Z}$

$$X_L = X_L$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

(3)

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

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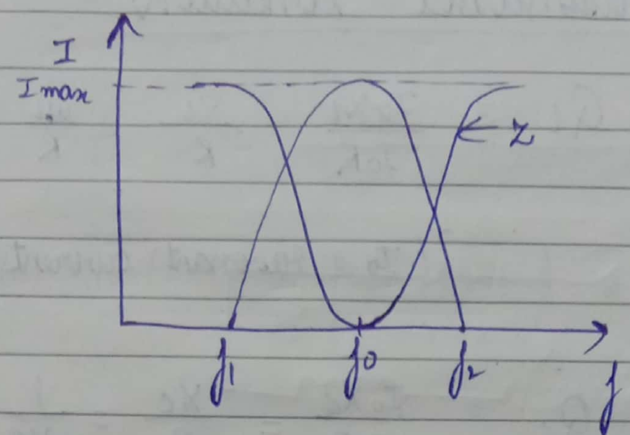

f_1 & f_2 : half power frequency

Lower half power ν

Higher half power ν

<u>Q factor at resonance:</u>

$$Q = \frac{V_L}{V} = \frac{V_C}{V}$$

When not in resonance condition

$$Q_L = \frac{\cancel{I_L} \times \cancel{R}}{\cancel{I} \times \cancel{R}} \quad \frac{I_L \times X_L}{I \times R} = \frac{I_L X_L}{IR}$$

$$Q_C = \frac{I_C \times X_C}{I \times R} = \frac{I_C X_C}{IR}$$

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$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{C\omega_0 R}}$$

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In resonance condition:

$$Q_L = \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R} = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

(I_0 = resonant current)

$$Q_C = \frac{I_0 X_C}{I_0 R} = \frac{X_C}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

(I_0 = resonant current)

✓ Bandwidth of series resonating ckt →

at resonance : $X_L = X_C$

at half power " :

$$X = \pm(X_L - X_C) = R$$

higher पर X_L exceed करेगा

lower " X_C " करेगा

for f_2, $\left(\omega_2 L - \frac{1}{\omega_2 C}\right) = R$ —— ①

for f_1, $\left(\omega_1 L - \frac{1}{\omega_1 C}\right) = -R$ —— ②

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(Book में kaafi है पर जितना class mein करवाया है उतना ही करना)

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Adding ① & ②,

$$(\omega_2+\omega_1)L - \frac{1}{C}\left(\frac{\omega_1+\omega_2}{\omega_1\omega_2}\right) = 0$$

$$L = \frac{1}{C}\left(\frac{1}{\omega_1\omega_2}\right)$$

$$\boxed{\omega_1\omega_2 = \frac{1}{LC}} \quad \text{— ③}$$

Subtracting ① & ② and divide by L:

then we'll get

$$\boxed{(\omega_2-\omega_1) = \frac{R}{L}} \quad \text{— ④}$$

$$Q = \frac{\omega_0 L}{R} \quad \star$$

$$\boxed{\frac{R}{L} = \frac{\omega_0}{Q}} \quad \text{— ⑤}$$

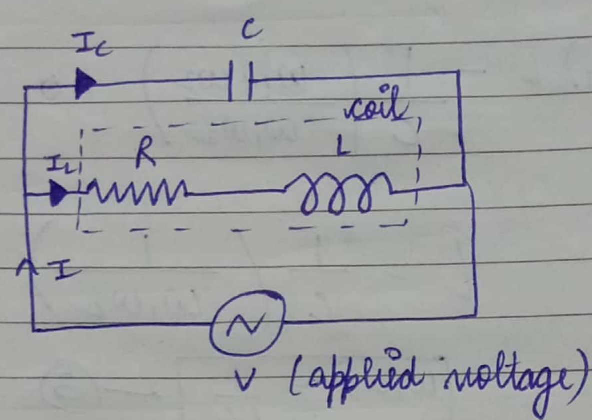
from eq^n — ④

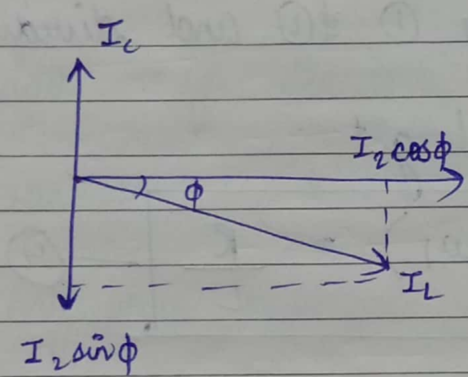
$$\omega_2-\omega_1 = \frac{\omega_0}{Q}$$

$$\boxed{Q = \frac{\omega_0}{\omega_2-\omega_1}} \boxed{= \frac{f_0}{f_2-f_1}} \boxed{= \frac{\text{resonant } \nu}{\text{Bandwidth}}}$$

Imp. ⊙ Derivation of calculating Bandwidth

Parallel resonance:





$\boxed{I = I_L \cos\phi}$

★ Resonance, तब आएगी जब imaginary part zero होगा

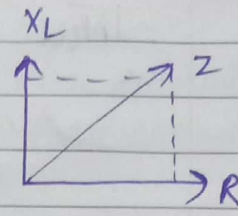
∴ at resonance,

$$I_C = I_L \sin\phi$$

$$\frac{V}{X_C} = \frac{V}{(R+jX_L)} \times \frac{X_L}{Z_L}$$

$$\boxed{R+jX_L = Z_L}$$

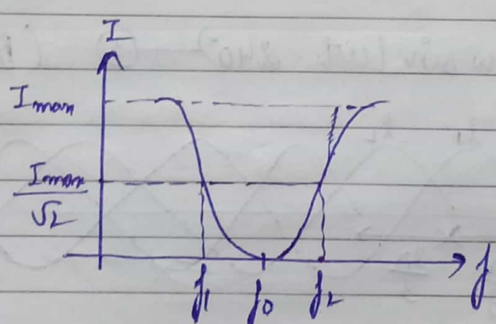
$$\boxed{\frac{V}{X_C} = \frac{V}{Z_L} \times \frac{X_L}{Z_L}}$$



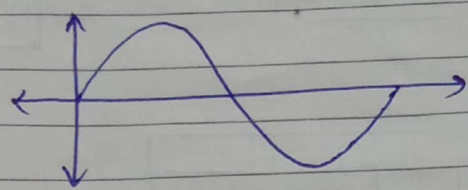
$$\boxed{f_0 = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}}$$

$$\frac{V}{Z_\Omega} = \frac{V}{Z_L} \times \frac{R}{Z_L}$$

$$\boxed{Z_\Omega = \frac{L}{CR}}$$



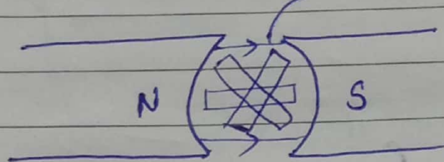
Three phase signals (for uninterrupted power supply)



single phase

→ Generation :

separated at 120°

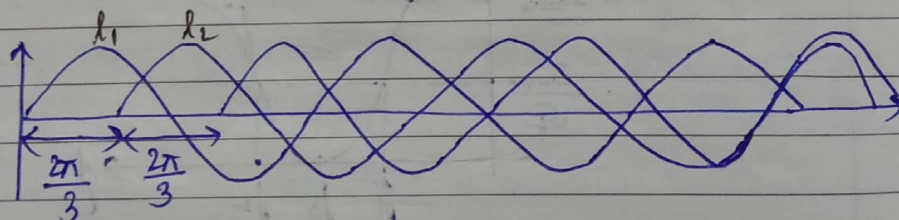


$$e = -\frac{d\phi}{dt}$$

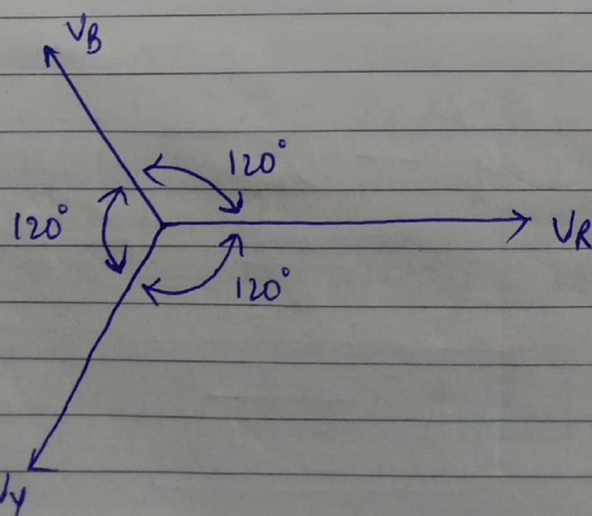
$e_1 = E_m \sin \omega t$ —— ① (Red colour)

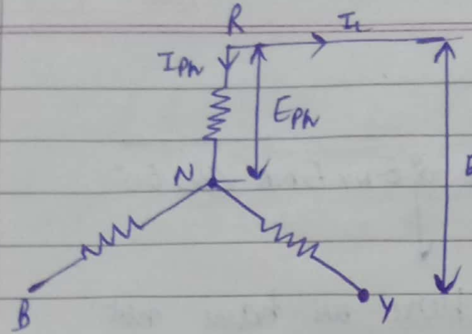
$e_2 = E_m \sin(\omega t - 120^\circ)$ —— ② (Yellow colour)

$e_3 = E_m \sin(\omega t - 240^\circ)$ —— ③ (Blue colour)

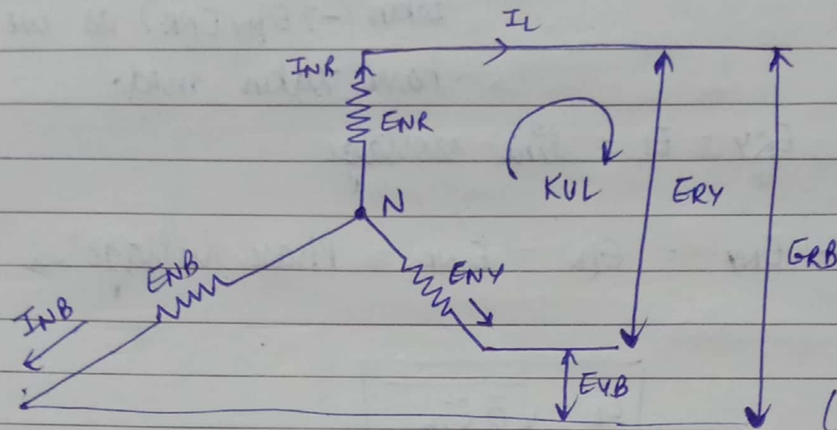


→





I_L = line current = I_{NR}
⇒ Phase current

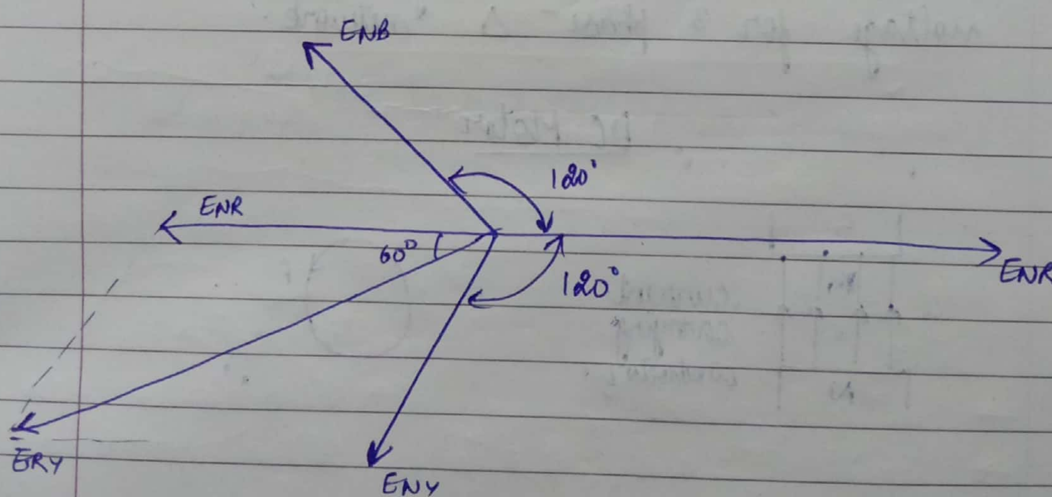


(all parts are identical)

$$\vec{E}_{NR} + \vec{E}_{RY} - \vec{E}_{NY} = 0$$

$$\vec{E}_{NR} + \vec{E}_{RY} = \vec{E}_{NY}$$

$$\vec{E}_{RY} = \vec{E}_{NY} - \vec{E}_{NR}$$



By ||gm law :

$$E_{RY} = \sqrt{E_{NY}^2 + E_{NR}^2 + 2E_{NY}E_{NR}\cos 60^\circ}$$

(Here we have not taken $(-2E_{NY}E_{NR})$ as we have taken mod.

$E_{RY} = E_L =$ line voltage

if given points are identical ← $E_{NY} = E_{Ph} = E_{NR} =$ Phase voltage

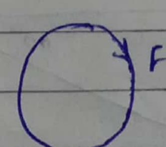
$$\boxed{E_L = \sqrt{3E_{Ph}^2}}$$

$$\boxed{E_L = \sqrt{3}\, E_{Ph}}$$

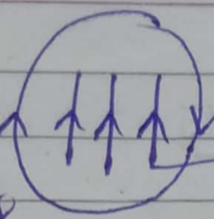
Q find relationship ① line current and phase current, ② line voltage and phase voltage for 3 phase Δ network.

DC Motor

current carrying conductor



F more as support



F (higher to lower)

$F = BIL$ (FLHR)

$E_b = \frac{\phi P N Z}{60 A}$

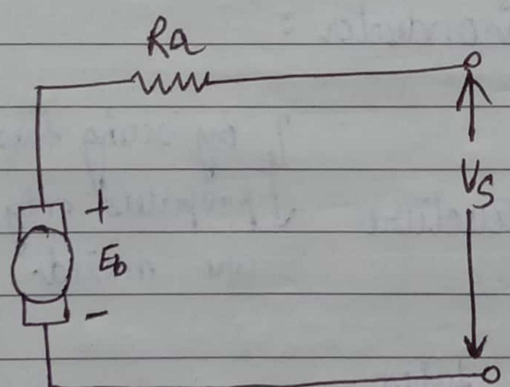
Back emf

ϕ = flux

P = no. of poles

N = speed of rotation of conductor

A = no. of parallel path



(I_a = armature current)

$V_S = E_b + I_a R_a + V_{brush}$